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AMERICAN

MATHEMATICAL SOCIETY

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A HISTORICAL AND CRITICAL REVIEW OF MATHEMATICAL SCIENCE.

EDITED BY

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BULLETIN OF THE

AMERICAN MATHEMATICAL SOCIETY

THE THIRD SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Third Summer Meeting of the American Mathematical Society was held in the Lecture Hall of the Society of Natural Sciences at Buffalo, N. Y., on Monday, August 31, and Tuesday, September 1. The Society continued for the present year the policy of affiliation, as regards the Summer Meeting, with the American Association for the Advancement of Science. In deference to the desire of the latter organization, the meeting of the Society was held the week following that of the Association. The register of attendance and the close and undivided attention given to the work of the meeting show that this arrangement was, on the whole, profitable for the Society.

Among those in attendance were: Dr. E. M. Blake, Professor Maxime Bôcher, Mr. J. M. Brooks, Professor F. N. Cole, Professor J. E. Davies, Professor E. W. Davis, Professor A. T. De Lury, Dr. L. E. Dickson, Professor W. P. Durfee, Professor H. T. Eddy, Professor T. S. Fiske, Miss Ida Griffiths, Dr. G. W. Hill, Dr. J. E. Hill, Professor T. F. Holgate, Dr. J. I. Hutchinson, Professor E. W. Hyde, Mr. P. A. Lambert, Dr. G. H. Ling, Professor James McMahon, Professor Mansfield Merriman, Professor E. H. Moore, Professor W. F. Osgood, Professor J. P. Pierpont, Professor Oscar Schmiedel, Dr. Virgil Snyder, Mr. W. M. Strong, Professor L. G. Weld, Professor H. S. White, Professor C. B. Williams, Miss E. C. Williams, Miss M. F. Winston, Professor F. S. Woods and Professor Alexander Ziwet.

Two sessions were held each day, at 10 A. M. and 2.30 P. M. respectively. The President, Dr. G. W. Hill, occupied the chair. On the recommendation of the Council Professor O. W. Anthony, New Windsor College, New Windsor, Md.; Mr. F. H. Clutz, Johns Hopkins University,

Baltimore, Md., and Miss E. K. Wentz, Indianapolis, Ind., were elected to membership. Ten nominations for membership were received.

The President read to the Society a communicatiom from the committee in charge of the susquicentennial celebration of the founding of Princeton University, inviting the members of the Society to attend the courses of lectures to be given at Princeton in the week preceding the celebration by Professor Felix Klein and Professor J. J. Thomson.

The following papers were read:

(1) Methods of defining monogenic functions. Dr. E. M. BLAKE.

(2) An existence theorem for a class of linear euthymorphic functions of a single variable. Dr. E. M. BLAKE.

(3) A graphic method for the study of uniform convergence. Professor W. F. OSGOOD.

(4) Uniform convergence and the integration of series term by term. Professor W. F. OSGOOD.

(5) Two triply-infinite systems of simple groups. Dr. L. E. DICKSON.

(6) Ternary algebras. Professor J. B. SHAW.

(7) The geometry upon three surfaces of the seventh order. Dr. J. E. HILL.

(8) A special form of a quartic surface. Dr. J. I. HUTCHINSON.

(9) Note on integral and integro-geometric series. Professor E. D. ROE.

(10) The cross-ratio group of $n!\,(n-3)$-*ic* Cremona transformations of flat space of $n-3$ dimensions. Professor E. H. MOORE.

(11) Criteria for the reality of nodes in Dupin's cyclides, with a corresponding classification. Dr. VIRGIL SNYDER.

(12) Numerically regular reticulations upon surfaces of deficiency higher than 1. Professor H. S. WHITE.

(13) Loci of the equations $p=\varphi^{z}e$ and $p=\varphi^{z}\zeta^{y}e$. Professor E. W. HYDE.

(14) On the hypothesis of the successive transmission of gravity, and the possible perturbative effect on the earth's orbit. Professor JAMES MCMAHON.

(15) The continuity of chance. Professor E. W. DAVIS.

(16) A method of finding, without a table, the number corresponding to a given logarithm. Dr. ARTEMAS MARTIN.

(17) Table giving the first forty roots of the Bessel equation $J_0(x)=0$, and the corresponding values of $J_1(x)$. Professors B. O. PEIRCE and R. W. WILLSON.

(18) On the projective group. Professor HENRY TABER.

The first of Dr. Blake's papers gives a classification of the methods which have been used for defining monogenic functions, such as algebraic, transcendental, differential and functional equations, assigned singularities, representability upon a given Riemann's surface, etc. Functions φ, defined

by an equation $G\left[z, \varphi(z), \varphi\left(\frac{a_1 z+\beta_1}{\gamma_1 z+\delta_1}\right), \cdots \varphi\left(\frac{a_n z+\beta_n}{\gamma_n z+\delta_n}\right)\right]=0,$

where G is algebraic, are suggested as a field which should yield important results. Dr. Blake's second paper contains a proof of the theorem that the multipunctual equation

$$\varphi(z)+\sum_1^n p_\nu(z)\cdot\varphi\left(\frac{a_\nu z}{\gamma_\nu z+1}\right)=0,$$

where $$|a_\nu|<1\,;\ 1+\sum_1^n p_\nu(0)=0\,,$$

and $$1+\sum_1^n p_\nu(0)\,a_\nu^m \neq 0,\ (m=1,2,\cdots)$$

possesses (for any arbitrary selection of branches of $p_\nu(z)$, are holomorphic within the domain of $z=0$) one and only one solution $\tau(z)$ which is holomorphic in the domain of $z=0$, and has at that point the value 1. Under the same conditions $\tau(z)$ multiplied by a constant is a solution, and these are the only solutions which are holomorphic at the point $z=0$.

Professor Osgood's first paper will be published in full in the BULLETIN. His second paper, which is to appear in the *American Journal of Mathematics*, is divided into two parts.

In Part I. the most general manner of the convergence of a function $s_n(x)$ satisfying the following conditions is studied by the aid of the geometric method set forth in the foregoing paper: (1) $s_n(x)$ is a continuous function of the real variable x; (2) when x is fixed and n increases without limit, $s_n(x)$ approaches a limit, $f(x)$; (3) $f(x)$ is continuous. Let x_0 be any value of x within the interval of convergence. Three cases can arise: a set of values $x', x'', \cdots$ with $\lim_{i=\infty} x^{(i)}=x_0$ and a corresponding set $n'\ n''\cdots\cdots$ with $\lim_{i=\infty} n^{(i)}=\infty$ can be so determined that (a) $\lim_{i=\infty}|s_{n^{(i)}}(x^{(i)})|=\infty$; in this case x_0 shall be distinguished as a X-point; (b) $\lim_{i=\infty}|s_{n^{(i)}}(x^{(i)})|\neq f(x_0)$, but perhaps cannot be made infinite;

in this case x_0 shall be designated as a ξ-point; the set of ξ-points thus includes the set of X-points; (c)

$$\lim_{i=\infty} s_{n^{(i)}}(x^{(i)}) = f(x_0),$$

no matter how the above set $(x^{(i)}, n^{(i)})$ be chosen; in this case x_0 is designated as a 3-point. It is shown that the X-points are dense throughout no interval and that the 3-points are dense and non-enumerable in every interval. The ξ-points may be dense throughout an interval, but cannot form a continuum. The problem of determining what sets of points can serve as ξ- resp. 3-points for some $s_n(x)$ is reduced to a problem in the Theory of Sets of Points.

Part II. is devoted to the question: When is

$$\int_0^{x_1} \left[\lim_{n=\infty} s_n(x)\right] dx = \lim_{n=\infty} \left[\int_{x_0}^{x_1} s_n(x)\, dx\right] ?$$

a question in Double Limits. If $s_n(x) = u_1(x) + \cdots + u_n(x)$, then this is equivalent to the question: When can the series $f(x) = u_1(x) + u_2(x) + \cdots$ be integrated term by term? It is found that such integration is always allowable if the interval (x_0, x_1) is free from X-points. If this is not the case,

$$\lim_{n=\infty} \left[\int_{x_0}^{x} s_n(x)\, dx\right],$$

$x_0 \leq x \leq x_1$, may be discontinuous; but it is shown by an example that it can also happen that this function is continuous and still never equal to

$$\int_{x_0}^{x} \left[\lim_{n=\infty} s_n(x)\right] dx, \text{ i.e., } \int_{x_0}^{x} f(x)\, dx, \text{ if } x \neq x_0;$$

in other words, it is possible that the term-by-term integral of a series of continuous functions, the value of the given series being a continuous function, may be a continuous function not equal to the integral of the given series. A sufficient condition is deduced that this should not be the case. These theorems give the most general conditions known to the author for the integration of a series term-by-term.

Dr. Dickson's paper contains a generalization for the Galois Field of order p^n, of the system of simple groups obtained by Jordan in the decomposition of the general linear group on m indices and of the group of the Abelian substitu-

tions on $2m$ indices. Two systems of simple groups are obtained whose orders depend on the three independent parameters m, n and p. Hermite's theorem on the analytical representation of substitutions is also generalized for the Galois Field, and the result employed to make a complete determination of quantics of degree ≤ 6 suitable to represent substitutions on a power of p letters. Two remarkable classes of quantics of general degree are found, which possess an infinite sweep of suitability for representing substitutions.

Professor J. B. Shaw's paper deals with the general theory of all algebras of three units. After the introduction of the necessary definitions, those algebras are first considered in which one 'factor' only is necessary to determine the product. This part of the paper is essentially a resumé of the theory of the linear vector operator or nomon. Those algebras are then discussed in which the 'product' of *two* multiple quantities is necessary to determine the third. Special classes are considered and also associative algebras. At the end the portion of Lie's continuous groups in this theory is pointed out. In the absence of Professor Shaw his paper was read by Professor Hyde.

In his paper on the geometry upon three surfaces of the seventh order, Dr. J. E. Hill passes by the aid of the general cubo-cubic transformation, in three cases of which the principal systems of the two spaces degenerate, from a general cubic surface to a septic surface. The first surface is distinguished by three triple and three double lines, the second by a triple line and a double quintic, and the third by a triple conic and a double quartic of the second kind. Surface $S_{1^3, 1^3, 1^3, 1^2, 1^2, 1^2}$ possesses 9 simple lines, 36 conics, 9 sheaves and 126 isolated examples of plane cubics, 12 sheaves and 126 isolated examples of plane quartics, 3 sheaves and 21 isolated examples of plane quintics and 9 sheaves of plane sextics. Upon this surface there are also 9 sheaves and 126 isolated examples of twisted cubics; 1 net, 126 sheaves and 162 isolated examples of twisted quartics of the second kind and 9 sheaves of twisted quartics of the first kind; 84 nets, 504 sheaves and 513 isolated examples of twisted quintics, $d = 0$, and 36 nets and 36 sheaves of twisted quintics, $d = 1$; 36 webs, 630 nets, 1,332 sheaves and 756 isolated examples of twisted sextics, $d = 0$; 84 webs, 288 nets and 126 sheaves of twisted sextics, $d = 1$, and 9 webs of twisted sextics, $d = 2$. There are 6 sheaves and 21 isolated examples of tritangent planes and 195 examples of quartitangent planes possessed by this surface, besides some special planes of much higher singularity of contact.

In the same way the surface $S_{3^3,5^2,1}$ is shown to possess 13 simple lines, 5 pairs of conic, 13 plane cubics, 1 sheaf of plane quartics, no plane quintics, and 13 sheaves of plane sextics. This surface still possesses 16 twisted cubics, 209 twisted quartics of the second kind, 16 sheaves and 208 isolated examples of twisted quintics, $d=0$, and 286 twisted quintics, $d=1$, beside twisted curves of higher orders. It has 312 tritangent planes, beside others of higher singularity of tangency.

The surface $S_{3^3,4^2}$ possesses 9 simple lines, 16 simple conics, no plane cubics, no plane quartics, 16 plane quintics, and 9 sheaves of plane sextics, 90 twisted cubics, 10 sheaves and 546 isolated examples of twisted quartics of the second kind, 126 twisted quartics of the first kind, 144 sheaves and 3,452 isolated examples of twisted quintics, $d=0$, 84 sheaves and 1,260 isolated examples of twisted quintics, $d=1$, and 180 twisted quintics, $d=2$, beside twisted curves of higher orders. This surface possesses 313 tritangent planes and one plane having 3-pointic contact along three sheets of the surface.

The special form of quartic surface considered by Dr. Hutchinson is the locus of the vertex of a cone passing through six given points. If these six points lie in involution on the twisted cubic through them, the coördinates of a point of the surface can be expressed in terms of elliptic functions. The special surface has two additional lines, each of which passes through a double point of the involution.

Professor Roe's paper is devoted to the determination of general formulæ for the summation of integral and integro-geometric series. The summation is affected by the employment of certain numbers $A_{r+\lambda}$, defined as the coefficients of $x^{r+\lambda}$ in the expansion of $(e^x-1)^r$. The numbers are expressible in determinant form, and their other properties are discussed in extenso. A complete general formula is given for the sum of n terms of an integro-geometric series. In the absence of Professor Roe, his paper was read by the Secretary.

Professor Moore's paper, which is intended for publication in the *Mathematische Annalen*, deals with the cross-ratios of n independent quantities z_i. All of these cross-ratios can be expressed rationally in terms of any $n-3$ independent ones among them. Starting with the fundamental system of $n-3$ ratios $[z_n\, z_{n-1}\, z_{n-2}\, z_i]$ $(i=1, 2, \cdots n-3)$, by permuting the z's we obtain in all $n!$ fundamental systems of the same type. The expression of these $n!$ systems in

terms of (any) one leads to the group $G_{n!}$ in question. For $n=4$ it is the well-known group of six linear fractional substitutions (collineations) generated by $\lambda' = \frac{1}{\lambda}$, $\lambda' = 1-\lambda$. For $n \geq 5$ the $G_{n!}$ is holoedrically isomorphic with the symmetric group in n letters. It contains a subgroup of $(n-1)!$ collineations permuting among themselves certain $n-1$ fundamental points; this $C^n G_{(n-1)!}$ was first given by Klein. The $G_{n!}$ arises by extending the $C^n G_{(n-1)!}$ by an inversion having $n-2$ of the fundamental points as critical points, and the remaining one as a fixed point.

Dr. Snyder defined a Dupin's cyclide, without the use of Line Geometry, by means of three simultaneous linear equations in Lie's hexaspherical coördinates. By associating these three equations with the point-complex and plane-complex, the reality of spheres common to the four complexes, given by the signs of determinants in the coefficients, indicate the presence of nodes.

Professor White discussed the problem of determining the numerically regular reticulations on surfaces of higher deficiency. For a convex surface (deficiency 0) Euler's equation, together with the requirement of regularity, gives three sets of finite integers for vertices, faces and edges of a polyhedron. These by duality become five, corresponding to the five "regular" polyhedrons. On a surface of deficiency >1 the modified equation of Euler, together with the same limitation of regularity, gives again a finite number of sets of finite integers for vertices, faces, and edges or lines. These sets are of two sorts: *derivative*, obtained from sets belonging to lower deficiencies; and *special*, not so obtainable, peculiar to the deficiency in question. These sets of characteristic numbers *can be realized on concrete models*. The author exhibited a set of models prepared by Mr. O. H. Basquin, comprising thirteen card and six plaster models illustrating deficiency 2, and seven card models showing the special sets for deficiency 3, and explained Mr. Basquin's method for obtaining derivative sets for any deficiency from those for lower deficiencies.

Professor Hyde discussed certain loci defined by the equation $p=\varphi^x e$ and $p=\varphi^x \psi^y e$, where p is a variable point, e a fixed point, x and y variable scalars, and φ and ψ linear point functions of the form $\varphi(q) = \Sigma(A_k e_k \cdot e_k | q)$, $(k=0,1,2$ or $k=0,1,2,3$, according as plane or solid space is under consideration). As x varies from $-\infty$ to $+\infty$ the first equation defines a curve described by p. This curve was discussed for different positions of e, the equation of the tangent

plane, the velocity of p for any value of x, etc., were determined both in two and in three dimensional space. The second equation defines a surface, which under certain conditions is a *skew* surface wholly inside the tetrahedron of reference, under other conditions a *cone* similarly limited. The reciprocal curves and surfaces were briefly discussed, and also the loci of the equations $p = \varphi^n e$ and $p = \varphi^n \psi^n e$, where $u = f_1 x$ and $v = f_2 y$, the first equation giving with suitable values of f_1, φ, and e curves terminated by arbitrary points, and the other surface having corners at arbitrary points.

Professor McMahon presented an investigation of the following hypothesis. Suppose that the sun is moving in a straight line with velocity u and that the whole solar system shares this translatory motion. Suppose also that the gravitational influence issues continually from the sun in waves that move outward with velocity v (perhaps equal to the velocity of light), and that when any wave reaches the earth the latter is attracted toward the wave center or point of space from which the wave issued. This effective center of acceleration is at a distance from the sun which varies between the limits $ka(1-e)$ and $ka(1+e)$ where k is the ratio of u to v, a is the semiaxis major and e the eccentricity of the earth's orbit. Then the orbit of the earth relatively to the sun is that which would be due to a center of force that performs small oscillations about its mean position. The law of this oscillatory motion was determined, and the equations of acceleration of the earth in its orbit, along and perpendicular to the radius vector were corrected for this small disturbance. Appropriate solutions of the resulting differential equations were given as far as terms in ke^2. The most important perturbative terms were examined, and their effect on the orbit determined.

Professor Davis' paper embodied an examination of the idea of continuity leading to Dedekind's definition of a continuous row of points, the one-to-one correspondence between point row and number series, integers, fractions, incommensurables, transcendents, etc. No *law* can define the positions of *all* points on a line. A perfect distribution must contain as a part of itself a chance distribution. But there must be continuity; the mind requires it to explain discontinuity. There must also be chance to explain the discontinuity of law. One-to-one correspondence exists between the points on a finite and an infinite line, an infinite surface, infinite three-way and n-way space, all space in all time, all movements, all qualities, the universe throughout

all time. The examination of the idea of chance shows the the latter presupposes an infinitely refined agnosticism, the limit of all possible or conceivable increase of knowledge. The theory of the Unknowable, the Absolute, the Will and wills were cognized under this philosophical aspect.

The object of Dr. Martin's paper was to deduce from the ordinary logarithmic series by proper modifications rapidly converging series from which the number corresponding to any given logarithm may be computed without tables. Four forms of series were given available under different conditions.

Dr. R. W. Willson and Professor B. O. Peirce presented a table giving the first forty roots of the Bessel equation $J_0(x)=0$ and the corresponding values of $J_1(x)$. The first ten values of x for which $J_0(x)$ vanishes have been given to ten places of decimals by Meissel. The next thirty and the values of $J_1(x)$ corresponding to the first forty roots have been computed by the authors by means of Vega's ten-place tables of logarithms, except in the few cases where a greater number of places was necessary, and then recourse was had to Thoman's tables. The computation has been gone over twice.

Professor Taber's paper contains a theory of the special linear homogeneous group in n variables constituting a generalization of Study's theory of such a group for $n=2$. The results show that there are as many species of transformations of the special linear homogeneous group in n variables as there are factors of n. The paper also contains a theory of certain other sub-groups of the general projective group, analogous to Study's theory of the special linear homogeneous group of the plane.

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CELESTIAL MECHANICS.

Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac. Vols. V., VI., VII. Washington, 1894–5.

The progress made in modern times toward the practical solution of the problems of celestial mechanics has been in no way behind that of the more theoretical investigations. New and more efficient methods of attacking the problems

have been invented, greater rigour has been introduced and numerical results have been carried to a much higher degree of approximation than was even deemed possible in earlier days. The determinations of the constants from the large masses of scientific observations which are now available, extending over some two hundred years, have enabled astronomers to obtain data for the formation of numerical tables to represent the motions of the planets and satellites with an accuracy far beyond that originally required. This refined accuracy is, of course, one of scientific interest, and it may almost be said to have but one object in view—the verification of the Newtonian law of gravitation; at the present time the observed motions are referred to this cause alone. That a single law of this exact nature is sufficient to account for the complicated phenomena of the solar system, must be regarded as very remarkable in view of the approximate nature of most of the assumptions made for the grouping of other observed phenomena. The unexplained differences between the places of the heavenly bodies, as calculated by means of the Newtonian law and as deduced from observation, small as they are, are nevertheless, in some cases, well defined, and they appear to show the necessity either of some modification of the law of the inverse square or of the introduction of some unknown cause of disturbance or, perhaps, of the existence of some new force. The method adopted by Professor Newcomb in order to take account of these differences will be mentioned below.

It might seem at first sight that many suppositions might be made to account for these deviations, but a little reflection will show that this is not necessarily the case. A supposition may be made in order to explain certain motions of one or two bodies, unaccounted for by the Newtonian law, and its magnitude and general effect may be calculated. But, owing to the large number of bodies in the solar system, it is usually possible to find at least one on which the effect is much greater than the observations will admit, and the supposition must therefore be rejected. It will be seen, then, that the difficulty is not that of choosing out one of a number of possible causes of disturbance, but rather that of discovering one which will satisfy the observed phenomena in the motions under consideration, and at the same time not give rise to other phenomena which we know, from observation, cannot exist. If an excuse were needed for the refinements to which the numerical solutions of these problems are subjected, a sufficient one is contained in the re-

marks just made. The Newtonian law of gravitation is so wide in its application to all problems of physics that no efforts made towards a verification of its accuracy and a determination of its limitations will be useless.

In the volumes before us the American Nautical Almanac office has given a further proof of the efficiency and activity which has characterized it during the last twenty years. In a prefatory note to the first volume of the series, published in 1882, Professor Newcomb announced that 'the objects of the series of papers ..... are, a systematic determination of the constants of astronomy from the best existing data, a reinvestigation of the theories of the celestial motions, and the preparation of tables, formulæ and precepts for the construction of ephemerides and for other applications of the results.' A brief glance at the contents of the seven volumes published up to the present date will show how fully these objects have been carried out, and the names attached to the various memoirs will be a sufficient warrant for the permanent value of the results. Since the time of Leverrier it may be safely said that no government office has published astronomical results of such value, whether we view them from the side of pure science or in the more practical direction of the construction of ephemerides and tables. Most of the first four volumes, which I do not intend to discuss here, are concerned with the theoretical investigations of the celestial motions and with the determinations of the fundamental constants of astronomy from the existing data. The measurement of time, the velocity of light, catalogues of stars and the discussion of ancient eclipses have all received attention as necessary preliminaries to the accurate determination of the constants which refer more particularly to the motions of planets and satellites. In the later volumes we are given investigations of the perturbations of the planets, both periodic and secular, by new methods or by better and more complete applications of existing methods; the reduction of thousands of observations for the determinations of the elements; and the discussion and adjustment of these values so as to best represent the planetary motions. The last step is the formation of tables obtained from the results of theory alone, without those empirical corrections (with one exception), which the observations require, but which the theory does not seem to suggest.

The final stages of the work are now beginning to appear. The three volumes published during the last eighteen months contain, besides some of the concluding theoretical

investigations, new tables for the calculations of the positions of the Sun, Mercury, Venus, Jupiter and Saturn. For convenience, I shall put the memoirs into three divisions:

I. Papers on the Planetary Theory (Vol. V.).

II. Tables of the Planets (Vols. VI., VII.).

III. A Paper on the Lunar Theory (Vol. V.).

I. Of the papers on the planetary theory by Professor Newcomb, the first is entitled, *A Development of the Perturbative Function in cosines of multiples of the Mean Anomalies and of angles between the perihelia and common node and in powers of the Eccentricity and Mutual Inclination.* The main feature of the method of development is the use of symbolic operators, as explained in an earlier paper* by the same writer. In Vol. III. of the series it was applied to the development of the perturbative function in cosines and sines of multiples of the *eccentric* anomalies of the two planets. In general, however, the methods used for the determination of the planetary perturbations dèmand that the expansions be made according to multiples of the *mean* anomalies, and we must therefore obtain the values of the operators in the latter case. It is not necessary to state here the method of procedure; it rests mainly on the theorem, proved by Professor Newcomb in the papers referred to above, that the coefficients for any powers of the eccentricities e, e' can be found, when those for $e = 0$ and for $e' = 0$ have been found, by a simple multiplication of the resulting operators. The main part of the memoir is taken up with the algebraical values of the operators. The expressions are carried to the eighth order with respect to the eccentricities for the terms which are independent of the inclinations, and to the seventh order for the terms which depend on the square of the inclination. The continuation of the process, for any degree of accuracy with respect to powers of the eccentricities and inclination, is cast into the form of precepts for computation. It may be stated that the indeterminates which enter into each operator are, i, the order of the Lègendre's coefficient which corresponds to the part of the coefficient under consideration, and D, the derivative with regard to the logarithm of the ratio of the mean distances.

Before proceeding to the actual development the question of the limits within which they are convergent is considered. It is shown that when the inverse of the mutual distance between the planets is expanded in powers of σ^2

* *Amer. Jour. Math.*, Vol. III., p. 193.

(where σ is the sine of half the mutual inclination), the development is convergent provided

$$\sigma < \frac{r'-r}{2\sqrt{r\,r'}} \quad ,$$

r, r' being the heliocentric distances of the planets. This condition appears to be fulfilled for every pair of the eight principal planets. It may be noted that the satisfaction of this inequality gives little information concerning the stability of the solar system. The possible values which r, r' may take after the lapse of long periods of time, depend largely on the possibility of eliminating secular terms from their expressions and also on the convergence of the periodic series which are used to represent their values. The modification of the condition given above, when expansions in powers of the eccentricities are used, is deduced by considering the maximum value of the coefficient of any periodic term.

In the second paper of Vol. V., '*Inequalities of long period, and of the Second order as to the masses, in the mean longitudes of the four inner planets*,' Professor Newcomb computes the coefficients of certain periodic inequalities, the lengths of whose periods give rise to sensible coefficients. The inequalities considered are those arising from the consideration of the mutual actions of *three* planets and the sun, the perturbations arising from the mutual action of each *pair* of planets and the sun being supposed to be known. Owing to the immense number of possibly sensible inequalities, the first step is the examination of those combinations of the mean motions which can produce an inequality of long period. In other words, if the inequality be denoted by

$$A \cos \{(i\,n+i'\,n'+i''\,n'')\,t+a\},$$

where n, n', n'' are the mean motions of the three planets, i, i', i'' positive or negative integers, and A, a constants, it is necessary to know for what values of i, i', i'', the expression

$$i\,n+i'\,n'+i''\,n''$$

may be a quantity small in comparison with n, n' or n''. Tables of possible combinations, for different values of i, i' i'' satisfying this combination, are given. It is then necessary to find the corresponding coefficients.

A rough computation is first made in order to discover the numerical magnitudes of these coefficients. It is not

easy to feel satisfied with the results of this computation, especially on reading the appendix, in which it is stated that two inequalities discovered by von Haerdtl and published while the memoir was passing through the press, had been overlooked. In fact, the inequalities calculated have all been previously known to have sensible coefficients and the values of the latter are given by Leverrier in nearly all cases. Quite a different impression is, however, produced with respect to the calculations undertaken for the determination of those coefficients which are selected for accurate computation. For some of these, two methods were used and there can be little doubt as to their accuracy. Moreover, Leverrier's values, which are given for the sake of comparison, agree, with one notable exception, with those obtained by Professor Newcomb, although the methods used are entirely different; the differences do not exceed two-tenths of a second, so that the coefficients may be regarded as definitive. The exception mentioned is that inequality in the motion of Mars, whose argument has a motion

$$3J-8M+4E,$$

J, M, E, denoting the mean motions of Jupiter, Mars and Earth, respectively. The difference amounts to one-fifth of the coefficient of the sine of the argument, the value of which, as given by Professor Newcomb, is $-40''.8$. The period of this inequality being 1800 years, it is scarcely possible, at present, to test the two values by observational means. In the motion of the Earth, the coefficient is much smaller—about $6''.4$—and the discovery of it by Hansen caused an important change in the reckoning of the Sun's mean longitude.

In the fourth part of Vol. V., Professor Newcomb discusses the *Secular Variations of the Four Inner Plants.* For this purpose he uses the method of the Variation of Arbitrary Constants applied directly to the constant part of the perturbative function, in preference to the purely numerical method of Gauss. Instead of starting with the numerical value of this constant part, the perturbative function is developed algebraically in the first instance, and then, in order to extend the results through a period of centuries, it is only necessary to insert the values of the elements for the dates chosen and to perform the few additional steps which the integration of the equations for the elements involves. If the same accuracy is required with Gauss' method, the whole of the work must be repeated for each date. The

methods practically amount to finding the values arranged in powers of the time, each new date computed giving an additional term. Professor Newcomb observes that, in any case, the results are only true to the first order with respect to the masses, since the initial constant part of the perturbing function is alone used.

There is a theoretical objection to both of these methods which deserves some examination. In the construction of a planetary theory it is always desirable to have the expressions formed in such a way that only one set of numerical values shall be substituted for the arbitrary constants of the solution, *e. g.*, for the elements at a given epoch. Now the methods of procedure adopted to obtain exact values of the secular terms require that the elements at each epoch for which the variations are calculated shall be known. If we have recourse to observation to determine the elements at the different epochs the results are no longer deduced from theory alone, since we are then really using the observed values of the secular variations to obtain the values of the elements at dates differing from the standard epoch; and these secular variations are capable of being deduced from theory alone when the elements at the standard epoch are given. We may, however, compute the variations at the standard epoch, say 1850, and, by means of the values thus obtained, find approximate values of the elements at any other epoch. With these approximate values the variations at the second epoch may be computed; but it then becomes necessary to find what error has been made by taking the approximate instead of the true values of the elements at this date. A short calculation assures us that the error thus committed is of a very much higher order than the differences between the secular variations at the two dates. If, then, the observed values of the secular variations only differ from the calculated values by quantities of an order of magnitude small in comparison with the quantities sought, we may use indifferently the values of the elements as obtained by a discussion of the observations or by means of the calculated variations at the standard epoch.

In the present memoir Professor Newcomb calculates the variations at three separate dates, 1600, 1850, 2100, and for this purpose gives a table of the values of the elements at those dates. There is, however, no indication of the manner in which the elements at 1600, 2100 have been obtained (I assume that those at 1850 are deduced from the observations). From what has been said above it will be seen that this does not matter unless a large discordance exists

between the different possible determinations of the elements at 1600, 2100. The only large discordance is that between the observed and calculated motion of the perihelion of Mercury which amounts to 43″ per century—about $\frac{1}{13}$ of the whole; it appears, from a short examination of the values of this element at the three dates that the observed values have been used. It might seem that this large difference would make it somewhat difficult to discover how far the observations entered into the values of the corresponding secular variations. Fortunately, the differences between the three values, and therefore the terms in t^2 are so small that either the observed or the calculated value may be used without affecting the result with an error greater than that which will arise from such extended computations. I have dwelt at some length on this point, because one of chief objects in view is the comparison of theory with observation.

The development of the constant part of the perturbative function is made by the method referred to above. A high degree of accuracy is obtained by carrying nearly all terms to the eighth order with respect to the eccentricities and inclinations; occasionally induction is used to include terms of higher orders whenever there was a probability of the results being affected thereby. The usual canonical system of equations is then transformed by changing to variables which are analogous to those used by Hansen in his theories. Longitudes are reckoned along the orbits from departure points and the formulæ are reduced to the simplest possible forms. [In the middle of page 332, the equations for eccentricity and longitude should be

$$\frac{de}{dt} = -\frac{m'}{e\mu}\frac{a}{a'} n \cos\varphi \frac{dF}{dw} = -M_2' \frac{1}{e}\frac{dF}{dw},$$

$$e\frac{d\pi_1}{dt} = M_2' \frac{dF}{de} \qquad].$$

The results for the secular variations at the three epochs are finally obtained in forms given by the equations; the complete reduction is left until a discussion of the final values to be adopted for the masses has been made.* They

* This discussion will be found in Professor Newcomb's work, "Astronomical Constants," forming a supplement to the American Ephemeris for 1897. The final values obtained both by theory and observation for the secular variations are there given, together with an extended comparison and examination into the possible causes of the resulting differences.

are very satisfactory. The first differences between the three sets of values are in most cases constant within the limits of possible errors, showing that the terms in t^4 are negligible for a period which may be measured by several centuries.

II. Tables of the Planets.

Volume VI. contains tables for the calculation of the positions at any time of the four inner planets; of these three parts have appeared, the Tables of the Sun, of Mercury and of Venus, by Professor Newcomb; Volume VII. contains the Tables of Jupiter and Saturn, by Dr. G. W. Hill. The publication of these two volumes makes an epoch in the history of celestial mechanics which is perhaps of equal importance with that which marked the appearance of Leverrier's well-known work on the same subject. Founded on theories which have been specially worked out for the purpose, with new determinations of the constants from the additional data which have accumulated in large masses in the last thirty years, during which time observations have been made with an accuracy far greater than was previously possible, they can hardly fail to be of immense service, not only for the calculation of the annual ephemerides published by the various governments, but also for the detection of new causes of disturbance by the comparison of future observations with the tabulated places. Time alone can show to what extent they will represent the observed places, but we may confidently expect that many years will elapse before it becomes necessary to correct them to any appreciable extent.

The most important tables are naturally those of the Sun, and a little space may, perhaps, be devoted to the description of their general features. We first consider the values which have been adopted for the fundamental constants. Some of these (for example, the solar mean motion) have long been known with an accuracy which permits of little change in their values; others, especially those in which occur the perturbations (for example, the masses of certain of the planets) are yet very doubtful. The discussion of them is given in detail in Professor Newcomb's "Astronomical Constants," above referred to. It is impossible to consider here, even in a condensed form, the manner in which a balance is struck between the different values which the different classes of observations furnish. It may be mentioned that no less than 40,176 observations of the Sun alone have been reduced, and many thousands of further observations of the planets have also been included.

Of the constants whose values are most open to doubt, the solar parallax and the masses of the planets are the chief. Eight classes of observations are discussed for the determination of the former, and the values range from 8″.759 to 8″.857; the greatest weights are assigned to the values obtained from the constant of aberration.

The value adopted is:

Mean equatorial horizontal solar parallax = 8″.790.

The constant of aberration is 20″.501.

The adopted masses of the planets, with the Sun as unit, are:

Mercury,	1: 600,000
Venus,	1: 408,000
Earth + Moon,	1: 329,390
Mars,	1:3,093,500
Jupiter,	1: 1,047.35
Saturn,	1: 3,501.6
Uranus,	1: 22,756
Neptune,	1: 19,540

and Moon : Earth : : 1:81.45.

Of these, the mass of Mercury is extremely doubtful, and it may differ from the above by one-third, or even one-half, of its whole amount; Professor Newcomb considers the assumed value to be probably a little too large. The periodic perturbations produced in the motion of the earth by this planet are, however, so small—amounting to only two or three hundredths of a second in longitude—that any admissible change in its mass will scarcely affect the tables. The above values are used in all of the tables of Vol. VI.

The chief discordance between theory and observation is in the secular motions of the perihelia, especially in that of Mercury. The possible causes of this are discussed very fully in the "Astronomical Constants." The only one which Professor Newcomb considers at all probable is that the law of attraction is not that of the inverse square, but that the index differs from 2 by a small quantity (Hall's hypothesis). He shows that with the value 2.000 000 1574 the outstanding differences in the motions of the perihelia of Mercury and Mars can be accounted for and that the effects on the other planets are within the limits of possible errors. No other sensible changes will be produced. It remains to see what effect the new law will have on the motion of the lunar perigee. On comparing Hansen's theoretical value (the only one which aspires to the required degree of precision) with the observed value, the small annual difference

(1″.56) is almost completely accounted for. But Hansen's value for the motion of the node differs from the observed value by a quantity twice as large: thus, either doubt arises as to accuracy of Hansen's calculations, or else some other cause exists affecting the motion of the node, since the above value for the index will not produce any sensible effect on this motion. As the observed motions of the lunar perigee and node are known with considerable accuracy, the question, so far as the moon is concerned, awaits a more exact theoretical determination of these motions.

The differences between theory and observation in the secular variations of the perihelia, at least in the cases of Mercury and Venus, are so well established that it has seemed advisable to take them into account in the tables. This has been done in such a manner as to make them as accurate as possible without taking away their theoretical character. Professor Newcomb considers that the value of the index instead of being 2 is

$$2.000\ 000\ 161\ 20.$$

The only resulting corrections are the following additions to the secular motions of the perihelia per century: Mercury, 43″.37; Venus, 16″.98; Earth, 10″.45; Mars, 5″.55.

The introduction to each set of tables is divided into parts which severally contain the values of the constants and of the periodic and secular perturbations; the formation of the arguments and the plan on which the tables are constructed; a statement of the quantities used in each table; and finally, precepts for their use. The epoch adopted is 1900.0 Greenwich mean time. The periodic perturbations are added directly to the longitude, latitude and radius vector, the arguments being so formed that, in each table of double entry, one argument is constant throughout the table. The examples given show that scarcely any multiplications and few interpolations are needed, the majority of the operations consisting of simple additions; this result is obtained in the usual manner, by adding such constants to the numbers of each table that the quantities therein are always positive; the sum of the constants added is subtracted from the constant part of each coördinate.*

* It may be useful to note here that, in the introduction to the tables of the Sun, p. 18, last line but one, the constant part to be added to $\Delta \log \nu$ should be 1600 instead of 1500. The error does not, however, affect the tables or the precepts for their use; it would only arise should the small correction to $\log \nu$, there noted, be deemed necessary.

Some statistics will best show the extent of the tables and will give a general idea of the labor involved in the calculation of an ephemeris. Those of the Sun are 38 in number, of which 6 are necessary for the formation of the arguments, 11 for the true longitude, 10 for radius vector, 4 for the latitude and 7 for the calculation of the precession, nutation, etc.; the whole occupying 136 quarto pages. Only two interpolations with second differences appear to be necessary for the longitude and radius vector and one for the latitude. The number of figures to be written in the calculation of an isolated position is about 1000; this number would, of course, be very much less, for each position, in the formation of continuous ephemeris. The results are obtained to the hundredth of a second of arc in the longitude and latitude and to 8 places in the logarithm of the radius vector.

The tables of Mercury and Venus are 22 and 27 in number, occupying 82 and 92 pages respectively, while the computation of an isolated position requires the writing of about 600 figures in each case. The results are carried to the same degree of accuracy as in the case of the Sun, with the exception of the logarithm of the solar radius vector of Mercury, which is taken to 7 places.

The tables of Jupiter and Saturn in Vol. VII. bear the signature of Dr. Hill and they are formed on the results of the theory worked out by the same writer in Vol. IV. of the series. The method there adopted was that of Hansen, with some modifications. The difficulties in obtaining a solution of the problem which these two planets present, and the somewhat unsatisfactory nature of previous theories, have caused the existing tables to present differences from the observed values which cannot be accounted for by errors of observation and which are therefore referred, in the first place, to defects in the theory or in the values of the constants used. In the "New Theory of Jupiter and Saturn," Dr. Hill has adopted the numerical method with values of the elements which are approximate, but which may be supposed to differ from the true values by certain appreciable though small quantities.

In order to obtain more correct values of the elements, Dr. Hill has formed provisional tables from his theory and has compared them with abont 19,000 observations of the two planets, extending from 1750 to 1865; a little more than half of these being in right ascension and the rest in declination. Normal equations having been formed, the corrections to the values of the elements are obtained and

the necessary changes in the theory follow. The main results of this investigation are given in the introductions to the tables. The mass of Jupiter, which results from the perturbations of Saturn by this planet is $1 : 1047 . 378 \pm .121$, and the mass of Saturn, from the perturbations of Jupiter is $1 : 3502 . 20 \pm .53$. These are very close to the values adopted by Professor Newcomb (see above), and the latter are used in the tables.

The tables are not uniform with those of Volume VI. The epoch is taken to be 1850.0, G. M. T., while the masses of Uranus and Neptune differ slightly. Newcomb's correction to the motions of the perihelia is insensible in the cases of these planets. Further, the form of the tables, owing to the use of Hansen's method, is quite different. The latter are 68 in number for Jupiter and 71 for Saturn, but the space occupied by them, and the number of figures to be written for the calculation of an isolated position, are about the same. In order to ensure the accuracy required and at the same time not to increase the size of the tables by the use of an extra decimal place, Dr. Hill adopts a device of Leverrier. The periodic inequalities of the mean anomaly and of the logarithm of the radius vector have been multiplied by 3; the sums of the results when these are taken out are to be divided by the same number. The difficulty which it is required to avoid is brought about by the use of the decimal scale of notation, which gives a certain degree of accuracy for a given number of places of decimals, or an accuracy ten times as great by the use of another place. The accuracy required is one midway between these two, and this is obtained without adding another decimal place by the artifice just mentioned. Full details are given as to the construction of the tables, with specific directions for their use.

III. *Action of the Planets on the Moon.* Vol. V. Pt. III.

The problem involved in the investigation of the effects of planetary action on the motion of the Moon is generally regarded as one of the most difficult in the whole range of celestial mechanics. Besides the usual difficulty which always arises in the investigation of any part of the lunar motion, there is added another which occurs in the planetary theory, namely, the slow convergence of the series in the cases of those planets which are not far from the earth and the consequent effect on the coefficients whenever the periodic terms have long periods. Further, it is usual and frequently necessary in the lunar theory to avoid all terms in which the time occurs outside the signs, sine and cosine

and the number of combinations which the different arguments give rise to becomes very great. Until the last few years the problem has been attacked chiefly from its practical side, and few theoretical results have been obtained. A first glance at the reading of Professor Newcomb's memoir seems to indicate a wide field for investigations into the theory; in any case the practical results, notwithstanding the labors of Hill, Radau, Newcomb and of the earlier investigators, leave much to be desired. The problem is in reality a case of the disturbed motion of three bodies, rather than a further extension of the problem of two bodies, and it is from this point of view that Professor Newcomb attacks it in the memoir under consideration here.

The paper opens with a more rigorous investigation of the results of the lunar theory when the disturbing action of the Sun is alone considered. The general equations of the motions of the bodies are first found and reduced to six of the second order, rectangular coördinates being used. Three of these equations refer to the motion of the Moon with respect to the Earth, and the other three to that of the Sun with respect to the centre of the mass of the Earth and Moon, the bodies being considered throughout as spheres. The method of solving these equations is well known. The Sun is first supposed to move in an elliptic orbit, and with this approximation a first approximation to the motion of the Moon is found; the result is contained in Delaunay's 'Théorie de la Lune.' Returning with these values of the lunar coördinates to the equations for the solar motion, more approximate values of the solar coördinates are found, and thence, by considering again the lunar equations, a second approximation to the lunar coördinates. Owing to the minuteness of the new terms furnished by the second approximations, we are enabled to stop at this point and to consider the problem of three bodies, so far as it refers to the Earth, Moon and Sun, as solved.

Professor Newcomb now introduces a fourth body and considers the effect of its disturbing action on the motions of the other three bodies. It is at this point that he makes a departure from the plan adopted by previous investigators. The latter have usually considered these perturbing effects separately, that is to say, they have found separately the perturbations produced on the Moon, those produced on the apparent motion of the Sun and, as a further step, the effects of the resulting perturbations of the Sun on the motion of the Moon. The method adopted to obtain them is that

of making, successively, the six elements of each motion vary in such a manner that the new terms may be included. Professor Newcomb considers the *twelve* elements of the Moon and Sun as variable and he thus includes at once all the three sets of disturbances; his method is therefore the disturbed motion of *three* bodies, and the new results which he obtains are mainly due to this plan of considering the perturbations.

The treatment is, at the outset, a general application of Lagrange's method for the variation of arbitrary constants. Representing the twelve arbitraries by $a_1, a_2, \cdots a_{12}$, the chief labor involved is the calculation of the Lagrangian coefficients (a_i, a_j)—the coefficients of the derivatives of $a_1, a_2 \cdots$ with respect to the time. The fact that these coefficients do not contain the time explicitly is the main factor in the determination of their values.

The six rectangular coördinates of the Moon referred to the Earth, and of the Sun referred to the centre of mass, of the Earth and Moon are known to be expressible in the forms,

$$x \text{ or } x' = \Sigma k \cos N,$$
$$y \text{ or } y' = \Sigma k \sin N,$$
$$z \text{ or } z' = \Sigma k \sin N',$$

where $$N \text{ or } N' = i_1 \lambda_1 + i_2 \lambda_2 + \cdots + i_6 \lambda_6;$$

in which, for N,

$$i_1 + i_2 + \cdots + i_6 = 0,$$

and for N'

$$i_1 + i_2 + \cdots + i_6 = 1.$$

Also, $$\lambda_i = l_i + b_i t.$$

In these expressions $l_1, l_2 \cdots l_6$ are six of the arbitrary constants of the solution, the coefficients k of the periodic terms and b_i of the time in the angles, being functions of six other arbitrary constants. The time thus enters into the expressions for the coördinates only through the angles λ_i in the manner just stated.

It is now possible to undertake the calculation of the Lagrangian coefficients (a_i, a_j). Six of the arbitraries are considered to be the l_i and the other six, $a, e, \gamma, a', e', \gamma'$, of which the k and b_i are functions. These coefficients are divided into parts according to the way in which they are derived, certain terms which are known to be quite insensible being neglected. The result obtained is as follows: Half

of the whole number of Lagrangian coefficients are zero, and the rest can be expressed as the partial derivatives of six functions $c_1\, c_2 \cdots c_6$ with respect to $a, e, \gamma, a', e', \gamma'$, the six functions containing only the latter six arbitraries. At this point, it is a simple step to obtain canonical equations for the variations of the elements. They are, in fact,

$$\frac{d c_i}{d t} = \frac{\partial R}{\partial l_i} \quad , \quad \frac{d l_i}{d t} = -\frac{\partial R}{\partial c_i} \quad , \quad (i = 1, 2, \cdots 6).$$

A few remarks on this result may, perhaps, be useful. It is known with respect to canonical systems of equations, that when half the arbitrary constants have been chosen, the form in which the rest of the arbitraries enter into the integrals is, in general, definite. It is natural to choose the l_i as six of the arbitrary constants, and the problem is therefore reduced to an investigation of the manner in which the other six arbitraries c_i enter into the expressions for the coordinates in order that the system may be canonical. The method used by Professor Newcomb to solve this problem, that is, to find the relations of the c_i to the coefficients k, b_i is powerful, if somewhat long, and one cannot but admire the manner in which the symbols are arranged and dealt with in order to bring about the desired result. Nevertheless it would seem possible to obtain it more easily by the use of the partial differential equation satisfied by the principal function. Jacobi has shown that if a solution of this partial differential equation, involving the coordinates q_i, the time, and half the required number of arbitrary constants can be obtained, the integrals of the problem are given by

$$p_i = \frac{\partial S}{\partial q_i} \quad , \quad l_i = \frac{\partial S}{\partial c_i} \quad ,$$

where S denotes the principal function, and p_i, q_i satisfy Hamilton's equations. The property which this method possesses is contained in the fact that the c_i and the l_i form a canonical system. In the present case, therefore, the problem reduces to the discovery of any solution of the equation for S, involving the coördinates, the time and the six arbitraries c_i. The difficulty involved in the discovery of such a solution generally makes it impossible to use the Jacobian method as a first means of investigation, and it is, in consequence, chiefly resorted to as a brief method of proving known results. No solution of the equation appears

to be readily available for the proof of Professor Newcomb's results.*

The kinetic energy of the system of three bodies is given by

$$2T=\mu_1(x^2+y^2+z^2)+\mu_2(x'^2+y'^2+z'^2)$$

where μ_1, μ_2 depend on the masses of the three bodies. The force function is Ω where

$$\Omega=\frac{m_1m_2}{\rho_{12}}+\frac{m_1m_3}{\rho_{13}}+\frac{m_2m_3}{\rho_{23}}$$

m_1, m_2, m_3 being the three masses. The function c_i, which is first to be considered as an arbitrary constant, is obtained by summing the products

$$k^2(i_1b_1+i_2b_2+\cdots+i_6b_6)i_i$$

for every periodic term in each of the six coördinates, multiplying those portions which arise from x, y, z by μ_1, and those portions which arise from x', y', z' by μ_2. This result, remarkable as it is from a theoretical point of view, is not of less interest practically, since it enables us to use canonical constants after solving the problem of three bodies and obtaining the values of the rectangular coördinates in terms of the constants of the solution.

The further result then follows that

$$b_i=-\frac{\partial T_0}{\partial c_i}$$

where T_0 is the non-periodic part of T when expressed in terms of the time as a sum of periodic terms. In the 'General Integrals of Planetary Motion,'† Professor Newcomb, has further shown that

$$T_0=-C,$$

where

$$T-\Omega=C.$$

Hence, when C has been expressed in terms of the c_i, the secular motions of the six angles are obtained by taking the partial derivatives of the constant of energy with respect to the c_i. This cannot, of course, be used to find them, since

* Since the above was written I have succeeded in proving the results obtained by Professor Newcomb in the manner indicated, and have also obtained some new formulæ which include the theorems of Adams mentioned below. These will be published shortly.

† Smithsonian Contributions to Knowledge, Vol. XXI.

the solution of the problem must be first obtained in order to find T_0 or C. A comparison of these results with those obtained by Adams concerning the relations of the secular motions of the lunar perigee and node, with the constant part of the lunar parallax,* is suggestive.

The next step is to obtain the values of the constants c_i from the known expressions for the coördinates of the Sun and Moon. The angles λ_i are defined to be the mean positions of the two bodies and of their perigees and nodes from some fixed line. When the relation of the theory to that of Delaunay is under consideration, this definition gives a simple relation between c_1, c_2, c_3 (which refer to the Moon) and the values of Delaunay's L, G, H after the last operation. Neglecting insensible terms, we obtain from the fact that a transformation of the kind known as "tangential" retains the canonical form of the equations, the relations

$$k_\epsilon = \mu_2 L, \quad k_\pi = \mu_2(G-L), \quad k_\theta = \mu_2(H-G),$$

where $k_\epsilon, k_\pi, k_\theta$, replacing c_1, c_2, c_3, denote the constants reciprocal to $\epsilon_0, \pi_0, \theta_0$ (the angles which give the mean positions of the Moon, its perigee and its node at time $t = 0$). The other parts of $k_\epsilon, k_\pi, k_\theta$, arising from the perturbations of the solar orbit on the lunar orbit, being negligible, $k_\epsilon, k_\pi, k_\theta$, might have been obtained from Delaunay's final values for L, G, H. Professor Newcomb, however, finds them directly by transforming Delaunay's final expressions for the longitude, latitude and parallax to rectangular coördinates and then computing them by the rules stated above. The results are expressed in terms of the final constants a, e, γ, a', e' used by Delaunay. He thus obtains an important verification of the accuracy of Delaunay's expressions. The values of L, G, H, given on pages 235, 236 of Vol. II. of Delaunay's work, are obtained in terms of his final constants by means of the formulæ given on page 800 of the same volume, and the results obtained by the two methods agree as far as the order to which they are carried. The labor of forming $k_\epsilon, k_\pi, k_\theta$ is great and it occupies a large part of Professor Newcomb's memoir. The chief parts of $k_{\epsilon'}, k_{\pi'}, k_{\theta'}$ are those due to the elliptic motion; the other portions due to the action of the Moon are very small, though not insensible.

The equations for the variations of the elements having been obtained, we arrive at formulæ which will enable us

*M. N. R. A. S., Vol. XXXVIII., pp. 460–472. See also a paper by the writer in the Amer. Jour. Math., Vol. XVII., pp. 318–358.

to calculate the effect of the action of a planet on these elements. The new part of the disturbing function is found in the usual way. The planetary action is usually divided into two parts, the *direct* action, arising from the attraction of the planet on the Moon, and the *indirect* action, arising from the attraction of the planet on the Earth and transmitted through the latter to the Moon. Professor Newcomb first considers the latter. Suppose the variations of the elements of the solar orbit have been obtained and enquiry be made as to the effect of these variations on the elements of the lunar orbit. For the latter we have, in general,

$$\frac{dk_\epsilon}{dt} = \frac{\partial R}{\partial \varepsilon}, \quad \frac{d\varepsilon}{dt} = -\frac{\partial R}{\partial k_\epsilon}$$

with two similar pairs of equations. The terms arising from R are those due to the direct actions of the planets. For the indirect actions we therefore have

$$\frac{dk_\epsilon}{dt} = 0, \quad \frac{d\varepsilon}{dt} = 0, \text{ etc.}$$

Now k_ϵ, k_π, k_θ are found in terms of a, e, γ, a', e', (γ' being neglected. If, then, we are considering the indirect action, and if, further, we suppose the perturbations of a', e' known, the perturbations of a, e, γ are obtained by solving the three equations

$$\frac{\partial k_\epsilon}{\partial a}\frac{da}{dt} + \frac{\partial k_\epsilon}{\partial e}\frac{de}{dt} + \frac{\partial k_\epsilon}{\partial \gamma}\frac{d\gamma}{dt} + \frac{\partial k_\epsilon}{\partial a'}\frac{da'}{dt} + \frac{\partial k_\epsilon}{\partial e'}\frac{de'}{dt} = 0,$$

$$\frac{\partial k_\pi}{\partial a}\frac{da}{dt} + \cdots\cdots = 0, \quad \frac{\partial k_\theta}{\partial a}\frac{da}{dt} + \cdots\cdots = 0.$$

If quantities of the first order, with respect to the perturbations of a', e', are alone considered, we finally arrive at the result that the indirect actions of the planets are obtained by solving the equations

$$\delta k_\epsilon = 0, \quad \delta k_\pi = 0, \quad \delta k_\theta = 0,$$

or, in Delaunay's notation,

$$\delta L = 0, \quad \delta G = 0, \quad \delta H = 0.$$

In other words, *if the coefficients in the time in the arguments**

* These must be expressed in the usual forms. Thus for the mean motion we put $\int n\,dt$ instead of nt.

and of the periodic terms in Delaunay's results were all expressed in terms of L, G, H, a', e', the perturbations due to the indirect actions of the planets would be obtained by merely inserting the variable instead of the constant values of a', e'.

The application of this remarkable result to the determination of the secular accelerations is immediate, since these inequalities depend on the variability of the solar eccentricity. It is only necessary to express Delaunay's final values of L, G, H in terms of his final constants, n or a, e, γ; the variations of a, e, γ are then obtained by solving the equations

$$\frac{\partial L}{\partial a}\delta a + \frac{\partial L}{\partial e}\delta e + \frac{\partial L}{\partial \gamma}\delta \gamma + \frac{\partial L}{\partial e'}\delta e' = 0,$$

$$\frac{\partial G}{\partial a}\delta a + \cdots = 0, \quad \frac{\partial H}{\partial e}\delta a + \cdots = 0,$$

with the given value of $\delta e'$. The secular accelerations are obtained by inserting these values for δa, δe, $\delta\gamma$, $\delta e'$ in $\int dt\delta n$, $\int dt\,\delta\pi_1$, $\int dt\,\delta\theta_1$ where n, π_1, θ_1 are the mean motions of the Moon and of its perigee and node. Professor Newcomb has verified this result in the case of the mean motion of the Moon. The application of the theorem to those periodic terms which are due to the indirect action of the planets is made in a similar way.

Professor Newcomb remarks that "this curious theorem may embody some principle applicable to the disturbed motion of three bodies which has not yet been fully mastered." It seems probable, from the way in which the result has been obtained, that it is a direct consequence of the use of canonical equations and of the form in which the time appears in the result. Canonical equations are the only ones at present known which will give symmetrical forms for researches into celestial mechanics, and the possibility of obtaining forms which can be made practically useful depends, to a large extent, on the fact that we have a system of constants (half the whole number required), ready to hand in the constant parts of the angles which occur under the signs, sine and cosine. The additional properties, first, that the force function does not contain the time explicitly, and secondly, that the coefficients of the time in the arguments can be expressed as the derivatives of the constant of energy, may indicate lines on which further investigations into the problem of three

bodies might be carried, especial attention being directed to the properties of canonical systems and especially of those of Delaunay and Newcomb.

Passing on to the consideration of the terms due to the direct action of the planets, Professor Newcomb obtains forms for the perturbations of the elements, with numerical coefficients similar to those obtained by Hill and Radau. He then considers the development of the perturbing function for planetary action after Hansen's method, and application is made to the cases of the principal inequalities arising from the various planets. Examination is also made into the possible cases where inequalities may become sensible owing to the presence of large divisors, the whole subject being dealt with in detail. There are, notwithstanding the researches made of late years into this subject, some outstanding inequalities of long period in the motion of the Moon which have not yet been satisfactorily accounted for by theory. I close with the remarks of Professor Newcomb on this subject. At the end of the Preface to his Memoir he says: "There are three possible sources to which may be attributed the apparent inequalities in question:

(α) Defects in the gravitational theory.

(β) Inequalities in the Earth's time of rotation.

(γ) Action on the Moon of other forces than the gravitation of known bodies.

The second of these causes seems to be not at all improbable, in view of recent discoveries respecting variations of latitude, which can only be accounted for by minute changes in the Earth's axis of rotation. But the independent tests of this cause offered by the observed Transits of Mercury do not show that it suffices to explain the observed inequalities in the motion of the Moon. Two other tests remain to be applied: The motion of the first satellite of Jupiter and observations of Mercury, Venus and Mars. So far as the evidence on these points has been brought out, the conclusion indicated is that inequalities in the Earth's rotation are real, but not large enough to produce the observed effects on the Moon's mean longitude. The subject is therefore still involved in doubt, and the first step toward the removal of this doubt is such a determination of the inequalities in the mean motion produced by the action of the planets as shall place the subject on an undoubted basis."

ERNEST W. BROWN.

HAVERFORD COLLEGE,
May 1, 1896.

NOTES.

THE regular October Meeting of the AMERICAN MATHEMATICAL SOCIETY, which was to be held on October 31st, will be omitted, and in its place a special meeting will be held at Princeton University on Saturday, October 17th, at quarter past three. Professor FELIX KLEIN, of Göttingen, and Professor J. J. THOMSON, of Cambridge, will both be present at this meeting and it is expected that they will address the Society.

PROFESSORS KLEIN and THOMSON, who are delegates to the sesquicentennial celebration of the founding of Princeton University, will each deliver a course of lectures at Princeton during the week preceding the celebration. Professor KLEIN will deliver four lectures on the *Theory of the Top* on Monday, Tuesday, Wednesday and Thursday, October 12th to 15th, at eleven o'clock A. M. Professor THOMSON will deliver four lectures on the *Discharge of Electricity in Gases* on Tuesday, Wednesday, Thursday and Friday, October 13th to 16th, at nine o'clock A. M.

THE forty-fifth annual meeting of the American Association for the Advancement of Science was held at Buffalo, August 24–29, President EDWARD D. COPE, of Philadelphia, presiding. The attendance was quite small, only 330 members being registered. In the absence of Professor W. E. STORY, Professor ALEXANDER MACFARLANE, of South Bethlehem, Pa., was elected Vice-President of Section A. The Secretary of Section A was Professor E. B. FROST, of Hanover, N. H.

The following papers were read before Section A :

(1) An analogue to DeMoivre's theorem in a plane point system. By Professor E. W. HYDE.

(2) Rational scalene triangles. By Dr. ARTEMAS MARTIN.

(3) New elements of the variable R Comae, resulting from observations in July and August, 1896. By Mr. HENRY M. PARKHURST.

(4) Photometric observations of colored stars. By Mr. HENRY M. PARKHURST.

(5) Motion of the great red spot and equatorial belt on the planet Jupiter from 1879 to 1896. By Professor G. W. HOUGH.

(6) On the direct application of a rational differential equation to a series of points whose coördinates represent

observed physical properties. By Professor ROBERT B. WARDER.

(7) A proposed fundamental integral-transcendent. By Professor J. MCMAHON.

(8) On the level of the sun-spots. By Professor EDWIN B. FROST.

(9) Sedenions. By Professor J. B. SHAW.

(10) On the distribution and the secular variation of terrestrial magnetism. No. IV: The component fields of the earth's magnetism. By Dr. L. A. BAUER.

(11) Determination of the weights of observations. By Professor J. R. EASTMAN.

(12) On the composition of simultaneous and successive vectors. By Professor ALEXANDER MACFARLANE.

The next annual meeting of the American Association will be held in Detroit during the week beginning August 9th. The meeting of the British Association will begin in Toronto on Wednesday of the following week, and it is expected that many of the members of the American Association will proceed to Toronto and join in welcoming the British Association to America. The President of the American Association for next year is Professor WOLCOTT GIBBS, of Cambridge. The officers of Section A for 1897 are: Vice-President, Professor W. W. BEMAN, of Ann Arbor, Secretary, Professor J. MCMAHON, of Ithaca.

As announced in the last number of the BULLETIN (Vol. II., p. 339), the German Mathematical Society held its annual meeting September 21–26, at Frankfort-on-the-Main, in conjunction with Section 1 (Mathematics and Astronomy) of the German Association of Naturalists and Physicians. The preliminary programme sent out at the end of August, contains the following list of papers:

(1) A. BRILL (Tübingen): "The resolution of a ternary form into linear factors."

(2) R. FRICKE (Braunschweig): "Groups of birational transformations."

(3) A. GUTZMER (Halle): Subject to be announced later.

(4) L. HEFFTER (Giessen): Subject to be announced later.

(5) F. KLEIN (Göttingen): "On the elliptic formulæ of the motion of a top."

(6) G. KOHN (Vienna): "On the geometric interpretation of the invariants of doubly binary forms."

(7) G. LANDSBERG (Heidelberg): Subject to be announced later.

(8) F. MEYER (Clausthal) "On full systems in trigonometry."

(9) VOGEL (Barmen): "On the values of trigonometric expansions within certain limits."

(10) H. SCHAPIRA (Heidelberg): "On homoidal functions."

(11) H. SCHAPIRA: "On a cribrum algebraicum, or the cofunctional generation of prime numbers."

(12) F. SCHILLING (Aachen): "On circular-arc-triangles with branch-points on the sides."

(13) L. SCHLESINGER (Berlin): "On the transformation group of a linear homogeneous differential equation."

(14) A. SCHOENFLIES (Göttingen): "Transfinite numbers, the axiom of Archimedes, and projective geometry."

(15) E. STUDY (Bonn): "On invariants of motion."

(16) E. STUDY: "The problem of Apollonius."

(17) A. TAUBER (Vienna): "On a partial differential equation of the second order."

(18) A. WANGERIN (Halle): "Contributions to the theory of the potential."

The following papers are announced for a joint session with Section 2 (Physics and Meteorology):

(19) H. BURKHARDT (Göttingen): "On vector analysis."

(20) HEUN (Berlin): "On the mathematical and mechanical principles in their application to technical problems."

(21) O. RAUSENBERGER (Frankfort): "The discontinuities of fluid motions."

Two papers will be read at a joint session with Section 11 (The teaching of mathematics and the natural sciences):

(22) ISRAEL-HOLTZWART (Frankfort): "Proposition for extending the intuitive methods of mathematical instruction."

(23) SCHWALBE (Berlin): "On the preparation of teachers of mathematics and natural science in higher institutions according to the requirements of modern times."

The programme also announces that the proposition of holding, in 1897, an international mathematical congress at Zürich will be discussed at the meeting.

THE recent meeting of the British Association for the Advancement of Science at Liverpool was one of the largest in its history, the total number in attendance nearly reaching 3,200. The presidential address by Sir Joseph Lister dwelt principally with the application of biology to medicine. Professor J. J. Thomson, President of Section A, delivered an address devoted largely to the Roentgen rays.

Both of these addresses have since appeared in *Science*. The date of next year's meeting at Toronto was fixed for August 18th. The President-elect is Sir John Evans.

FACULTÉ DES SCIENCES. During the first semester of the academic year 1896–1897 the following mathematical courses are offered at the Sorbonne in Paris: By Professor Darboux: The principles of geometry and the theory of triple systems of orthogonal surfaces.—By Professor Picard: Theory of algebraic functions of two independent variables and its applications to certain questions of the integral calculus.—By Professor Appell: General laws of Equilibrium and of motion.—By Professor Tisserand: Motions of rotation of celestial bodies in general and in particular those of the Earth and the Moon.—By Professor Poincaré: Electrodynamics.—By Professor Boussinesq: Study of internal friction of fluids with applications to continuous flow and to the gradual extinction of sound and liquid waves. The following supplementary courses are also given: By Mr. Painlevé: Differential and integral calculus.—By Mr. Koenigs: Kinematics of a solid body.—By Mr. Andoyer: The motion of translation of the planets by the method of variation of parameters.—By Mr. Raffy: Mathematical theories introductory to the study of physics. Mathematical conferences are conducted by Messrs. Raffy, Puiseux, Painlevé, Andoyer and Blutel.

UNIVERSITY OF BERLIN. The mathematical courses announced for the winter semester are the following: By Professor Frobenius: Theory of algebraic equations.—By Professor Fuchs. Elliptic functions; Linear differential equations.—By Professor Helmert: Least squares; The influence of masses upon the Earth's surface on the shape of the sea level.—By Professor Schwarz: Theory of surfaces and curves in space; Applications of elliptic functions with use of "Formeln und Lehrsätze;" Calculus of variations; Colloquium.—By Professor Hensel: Analytical geometry; Theory of numbers.—By Professor Hettner: Analytical treatment of surfaces of the second order.—By Professor Knoblauch: Differential calculus; Applications of elliptic functions.—By Professor Lehmann-Filhés: Least squares; Hansen's method for the computation of absolute perturbations.—By Dr. Hoppe: Analytical geometry; Differential calculus and theory of series.—By Dr. Kötter: Geometry of position; Partial differential equations with physical applications.—By Dr. Pringsheim: Kinetic theory of gases.—By Dr. Rubens: Fluid motion.—By Dr. Schlesinger: In-

tegral calculus; Theory of potential.—By Dr. Wien: Integration of differential equations presenting themselves in physics.

UNIVERSITY OF LEIPZIG. During the coming winter semester the following mathematical courses will be offered: By Professor Scheibner: Theory of entire functions; Higher arithmetic.—By Professor C. Neumann: General theory of functions; Discussion of mathematical problems.—By Professor Bruns: Celestial mechanics; Least squares.—By Professor Lie: Introduction to projective geometry; Differential invariants; Seminar.—By Professor Mayer: Analytical mechanics.—By Professor von Oettingen: Synthetic Geometry.—By Professor Engel: Introduction to differential and integral calculus; Ordinary differential equations.—By Dr. Hausdorff: Analytical geometry; Insurance.

AMONG the forthcoming books of The Macmillan Company we notice J. M. PAGE, "Differential equations;" W. B. SMITH, "Infinitesimal analysis;" H. D. THOMPSON, "Elementary solid geometry;" J. B. LOCK, "Trigonometry for beginners, revised for use in American schools and colleges by J. A. Miller;" HALL and KNIGHT, "Elementary trigonometry, revised for American use by F. L. Sevenoak;" F. CAJORI, "A history of elementary mathematics;" A. G. WEBSTER, "The mathematical theory of electricity and magnetism;" C. CHRISTIANSEN, "Theoretical physics, translated by W. F. Magie;" R. CLAUSIUS, "On heat, translated by C. H. Bierbaum and edited by R. C. Carpenter;" D. C. JACKSON, "Alternating currents and alternating current machinery."

PROFESSORS W. W. BEMAN and D. E. SMITH are preparing an authorized English edition of F. KLEIN'S "Vorträge über ausgewählte Fragen der Elementargeometrie," which will be published by Ginn & Co., of Boston, in the fall.

HENRI RESAL, a member of the French Academy of Sciences and for several years editor of the *Journal de Mathématiques pures et appliquées*, died on August 22d at the age of sixty-eight years.

DR. H. MINKOWSKI, professor of mathematics in the University of Königsberg, has been called to the Zürich Polytechnic Institute.

DR. GEORG SCHEFFERS, of the University of Leipsic, has been called to a professorship in the Polytechnic School at Darmstadt.

DR. ARNOLD MEYER, professor of mathematics in the University of Zürich, died at Zürich, July 7, 1896.

F. REULEAUX, professor of mechanical engineering in the Polytechnic School at Charlottenburg, near Berlin, has resigned his position. He has been engaged in teaching for 40 years.

W. VALENTINER, professor of astronomy in the Polytechnic School at Karlsruhe, has been called in the same capacity to the University of Heidelberg.

DR. L. CHRISTIAN WIENER, professor of descriptive geometry in the Polytechnic School at Karlsruhe, died July 31, at the age of 70.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AMODEO (F.). Lezioni di geometria projettiva; dettate nella R. Università di Napoli nell'anno scholastico 1895–96. Napoli, 1896. 8vo. 272 pp. Fr. 6.00

ANTOMARI (X.) et MALUSKI (A.). Manuel du baccalauréat: Mathématiques. Paris, 1896. 12mo. 368 pp. Cloth. Fr. 3.50

APOLLONIUS OF PERGA. Treatise on conic sections. Edited in modern notation, with introduction, including an essay on the earlier history of the subject, by T. L. Heath. London, Macmillan, 1896. 8vo. 170 and 254 pp. Cloth. $3.75

APPELL (P.) et LACOUR (E.). Principes de la théorie des fonctions elliptiques, et applications. Paris, Gauthier-Villars, 1896. 8vo. pp. 1–208. Subscription price for the whole work Fr. 12.00

AUTENHEIMER (F.). Elementary treatise on the differential and integral calculus. Russian translation by W. Göbel. Moscow, 1896. (Berlin, Friedländer.) 8vo. 4 and 300 pp. Mk. 5.00

BEUDON (J.). Sur les systèmes d'équations aux dérivées partielles dont les caractéristiques dépendent d'un nombre fini de paramètres. [Thèse.] Paris, Gauthier-Villars, 1896. 4to. 53 pp.

BILLY (J. DE). See FERMAT (P.).

BJÖRLING (C. F. E.). Lärobok i nyare Plan-Geometri. Lund, Gleerup, 1896. 8vo. 11 and 335 pp., 2 plates. Mk. 12.00

BOCKWOLDT (G.). Die analytische Geometrie in der Prima des Gymnasiums. Theil III. Neustadt, 1896. 8vo. 18 pp., 2 plates. Mk. 1.50

BOEHM (K.). Allgemeine Untersuchungen über die Reduction partieller Differentialgleichungen auf gewöhnliche Differentialgleichungen. Mit einer Anwendung auf die Theorie der Potentialgleichung. Leipzig, Teubner, 1896. 8vo. 3 and 58 pp. Mk. 2.00

BRAMBILLA (G.). Saggio di storia della ragioneria presso i popoli antichi. Milano, Boriglione, 1896. 8vo. 50 pp. Fr. 1.00

BÜRKLEN (O. T.). Formelsammlung und Repertorium der Mathematik, enthaltend die wichtigsten Formeln und Lehrsätze der Arithmetik, Algebra, niederen Analysis, ebenen Geometrie, Stereometrie, ebenen und sphärischen Trigonometrie, mathematischen Geographie, analytischen Geometrie der Ebene und des Raumes und der höheren Analysis. Leipzig, 1896. 12mo. 210 pp. Cloth. Mk. 0.80.

BUGÀIEV (N. V.). On the theory of functions and continued fractions. Moscow, 1896. (Berlin, Friedländer.) [Russian.] 8vo. 36 pp. Mk. 1.80

BUNKOFER (W.). Die arithmetischen Functionen der drei ersten Ordnungen. Wertheim, 1895. 4to. 18 pp. Mk. 1.50

BYERLY (W. E.). See HIGHER MATHEMATICS.

CAYLEY (A.). The collected mathematical papers. Vol. X. Cambridge University Press, 1896. (New York, Macmillan.) 4to. 14 and 616 pp. $5.50

COMBETTE (E.). Compléments du cours d'algèbre et notions de géométrie analytique, à l'usage des candidats á l'École spéciale militaire de Saint-Cyr et des élèves qui se préparent à suivre le cours de mathématiques spéciales. Paris, Alcan, 1896. 8vo. 288 pp. Fr. 4.00

COUTURAT (L.). L'infini mathématique. Paris, 1896. 8vo. Fr. 7.50

DANDOLO (G.). Intorno al numero; discussioni psicologiche. Padova, Draghi, 1896. 8vo. 85 pp. Fr. 1.75

DARBOUX (G.). Cours de géométrie de la Faculté des sciences: Leçons sur la théorie générale des surfaces et les applications géométriques du calcul infinitésimal. Vol. IV: Déformation infiniment petite et représentation sphérique. Paris, Gauthier-Villars, 1896. 8vo. 8 and 548 pp. Fr. 15.00

DRAGO (G.). Dottrina dei caratteri della divisibilitá. Torino, Clausen, 1896. 8vo. 31 pp. Fr. 2.00

FEDDERSEN (B. W.). See POGGENDORFF (J. C.).

FERMAT (P.). Oeuvres, publiées par les soins de P. Tannery et C. Henry, sous les auspices du ministère de l'instruction publique. Vol. III: Traductions par P. Tannery: (1) des écrits et fragments latins de Fermat, (2) de l'Inventum novum, de Jacques de Billy, (3) du Commercium epistolicum, de Wallis. Paris, Gauthier-Villars, 1896. 4to. 15 and 612 pp. Fr. 28.00

FICHTENHOLTZ (A.). See VASSILIEF (A.).

FISKE (T. S.). See HIGHER MATHEMATICS.

FLOQUET (G.). Sur certaines fonctions à trois déterminations considérées comme solutions d'une équation différentielle linéaire. Nancy, Berger-Levrault, 1896. 8vo. 11 pp.

FRIESENDORFF (T.) See MARKOFF (A.).

FRONTERA (G.). See SONNET (H.).

GASS (T.). See STURM (C.).

GÖBEL (W.). See AUTENHEIMER (F.).

GÜNTHER (S.). Jacob Ziegler, ein bayrischer Geograph und Mathematiker. Ansbach, 1896. 8vo. 64 pp. Mk. 2.00

HALSTED (G. B.). See HIGHER MATHEMATICS.

HANSEN (H.). Definitioner og Formler til Beregning af Legemernes Overflader og Rumfang. Kjöbenhavn, 1896. (Berlin, Friedländer.) 8vo. 8pp. [Lith.] Mk. 0.50

HAURE (M.). Recherches sur les points de Weierstrass d'une courbe plane algébrique. [Thèse.] Paris, Gauthier-Villars, 1896. 4to. 89 pp.

HEATH (T. L.). See APOLLONIUS OF PERGA.

HELLMANN (W.). Ueber die Anfänge des mathematischen Unterrichts an den Erfurter Schulen im 16. und 17. Jahrhundert und bis etwa 1774. Teil II. Erfurt, 1896. 4to. 16 pp. Mk. 1.20

HELMHOLTZ (H. VON). On the origin and meaning of the axioms of geometry. Russian translation. St. Petersburg, 1896. (Berlin, Friedländer.) 12mo. 58 pp. Mk. 1.20

HENRY (C.). See FERMAT (P.).

HIGHER MATHEMATICS. A text-book for classical and engineering colleges. Edited by M. Merriman and R. S. Woodward. (I. The solution of equations, by M. Merriman; II. Determinants, by L. G. Weld; III. Projective geometry, by G. B. Halsted; IV. Hyperbolic functions, by J. McMahon; V. Harmonic functions, by W. E. Byerly; VI. Functions of a complex variable, by T. S. Fiske; VII. Differential equations, by W. W. Johnson; VIII. Grassmann's space analysis, by E. W. Hyde; IX. Vector analysis and quaternions, by A. Macfarlane; X. Probability and theory of errors, by R. S. Woodward; XI. History of modern mathematics, by D. E. Smith.) New York, Wiley, 1896. 8vo. 12 and 576 pp. Cloth. $5.00

HYDE (E. W.). See HIGHER MATHEMATICS.

JAHRBUCH über die Fortschritte der Mathematik, begründet von C. Ohrtmann; herausgegeben von E. Lampe. Vol. XXV: Jahrgang 1893 und 1894. (In 3 Heften.) Heft 2. Berlin, Reimer, 1896. 8vo. pp. 853-1316. Mk. 11.00

JOHNSON (W. W.). See HIGHER MATHEMATICS.

KAMMER (A. ZUR). Integration einer linearen homogenen Differentialgleichung vierter Ordnung, deren Integral in der Horizontalfläche Unstetigkeiten zweiter Art besitzt. [Diss.] Kiel, 1895. 8vo. 54 pp.

KHEIL (C. P.). Ueber einige ältere Bearbeitungen des Buchhaltungstractates von Luca Pacioli. Prag, 1896. 8vo 8 and 128 pp. Facsimile. Mk. 4.00

KLEIN (F.). See MARKOFF (A.).

KREUZBERG (G.). Ein Beweis für die Zerlegbarkeit rationaler ganzer Funktionen einer Veränderlichen mit reellen Koefficienten in reelle Faktoren ersten und zweiten Grades. [Diss.] Bonn, 1896. 8vo. 32 pp.

LACOUR E.). See APPELL (P.) et LACOUR (E.).

LAMPE (E.). See JAHRBUCH.

LANG (P.). Ueber die Krümmungsverhältnisse der drei Scharen von Flächen zweiten Grades, die mit einem gegebenen ungleichaxigen Ellipsoide confocal sind. [Progr.] Kreuznach, 1896. 4to. 16 pp.

LEFEVRE (A.). Number and its algebra. Boston, Heath, 1896. 8vo. 219 pp. Cloth. $1.25

LILIENTHAL (R. VON). Grundlagen einer Krümmungslehre der Kurvenscharen. Leipzig, Teubner, 1896. 8vo. Mk. 6.00

LORIA (G.). Il passato e il presente delle principali teorie geometriche. 2a edizione accresciuta ed interamente rifatta. Torino, Clausen, 1896. 8vo. 20 and 346 pp. Fr. 8.00

LUCAS (E.). Récréations mathématiques. 2e édition. Vol. II: Qui perd gagne; les dominos; les marelles; le parquet; le casse-tête; les jeux de demoiselles; le jeu icosien d'Hamilton. Paris, Gauthier-Villars, 1896. 8vo. 247 pp.

MACÉ DE LÉPINAY (A.). Compléments d'algèbre et notions de géométrie analytique à l'usage des candidats à Saint-Cyr. 3e édition. Paris, 1896. 8vo. 482 pp. Fr. 5.00

MACFARLANE (A.) See HIGHER MATHEMATICS.

MALUSKI (A.). See ANTOMARI (X.).

MARKOFF (A.). Differenzenrechnung. Autorisierte deutsche Uebersetzung von T. Friesendorff und E. Prümm. Mit einem Vorwort von F. Klein. Leipzig, Teubner, 1896. 8vo. Mk. 6.00

MASSNY (W.). Ueber ebene Kurven, die bei circularer Inversion sich selbst zugeordnet sind. [Progr.] Beuthen, 1896. 4to. 14 pp. Mk. 1.00

MERRIMAN (M.). See HIGHER MATHEMATICS.

MODÈ (G.). Di una proprietà del dodecaedro e dell'icosaedro regolari convessi. Vicenza, Raschi, 1896. 8vo. 10 pp.

MÜLLER (C. F.). Henricus Grammateus und sein Algorismus de integris. [Progr.] Zwickau, Thost, 1896. 4to. 33 pp. Mk. 1.00

NETTO (E.). Vorlesungen über die höhere Algebra. (In 2 Bänden.) Vol. I. Leipzig, Teubner, 1896. 8vo. Mk. 10.00.

OCAGNE (M. D'). Cours de géométrie descriptive et de géométrie infinitésimale. Paris, Gauthier-Villars, 1896. 8vo. 11 and 429 pp. Fr. 13.00

OETTINGEN (A. J. VON). See POGGENDORFF (J. C.).

PACIOLI (L.). Divina proportione. Die Lehre vom goldenen Schnitt. Nach der venezianischen Ausgabe vom Jahre 1509 neu herausgegeben, übersetzt und erläutert von C. Winterberg. Wien, 1896. 8vo. 6 and 367 pp. Mk. 6.00

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